Chapter 6 Review

MULTIPLE CHOICE.

1. The following table gives the probabilities of various outcomes for a gambling game.

Outcome	Lose $1	Win $1	Win $2
Probability	0.6	0.25	0.15

What is the player's expected return on a bet of $1?
   a. $0.05 b. -$0.60 c. -$0.05 d. -$0.10
   e. You can't answer this question since this is not a complete probability distribution.

2. A binomial event has n = 60 trials. The probability of success on each trial is 0.4. Let X be the count of successes of the event during the 60 trials. What are μ_x and σ_x?
   a. 24, 3.79 b. 24, 14.4 c. 4.90, 3.79
   d. 4.90, 14.4 e. 2.4, 3.79

3. To use a normal approximation to the binomial, which of the following does *not* have to be true?
   a. $np \geq 5, n(p-1) \geq 5$ (or $np \geq 10, n(p-1) \geq 10$)
   b. The individual trials must be independent.
   c. The sample size in the problem must be too large to permit the problem on a calculator.
   d. For the binomial, the population size must be at least 10 times as large as the sample size.
   e. All of the above are true.

4. A 12-sided die has faces numbered from 1-12. Assuming the die is fair (that is, each face is equally likely to appear each time), which of the following would give the exact probability of getting at least 10 3s out of 50 rolls?
   a. $\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \cdots + \binom{50}{9}(0.083)^9(0.917)^{41}$
   b. $\binom{50}{11}(0.083)^{11}(0.917)^{39} + \binom{50}{12}(0.083)^{12}(0.917)^{38} + \cdots + \binom{50}{50}(0.083)^{50}(0.917)^0$
   c. $1 - \left[\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \cdots + \binom{50}{10}(0.083)^{10}(0.917)^{40}\right]$
   d. $1 - \left[\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \cdots + \binom{50}{9}(0.083)^9(0.917)^{41}\right]$
   e. $\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \cdots + \binom{50}{10}(0.083)^{10}(0.917)^{40}$

5. Which of the following is not a common characteristic of binomial and geometric experiments?
   a. There are exactly two possible outcomes: success or failure
   b. There is a random variable X that counts the number of successes
   c. Each trial is independent (knowledge about what happened on previous trials gives you no information about the current trial.)
   d. The probability of success stays the same from trial to trial.
   e. P(success) + P(failure) = 1

6. A school survey of students concerning which band to hire for the next school dance shows 70% of students in favor of hiring The Greasy Slugs. What is the approximate probability that, in a random sample of 200 students, at least 150 will favor hiring The Greasy Slugs?
   a. $\binom{200}{150}(0.7)^{150}(0.3)^{50}$ b. $\binom{200}{150}(0.3)^{150}(0.7)^{50}$
   c. $P\left(z > \frac{150-140}{\sqrt{200(0.7)(0.3)}}\right)$ d. $P\left(z > \frac{150-140}{\sqrt{150(0.7)(0.3)}}\right)$
   e. $P\left(z > \frac{140-150}{\sqrt{200(0.7)(0.3)}}\right)$

FREE RESPONSE

7. Find the μ_x and σ_x for the following discrete probability distribution:

X	2	3	4
P(X)	1/3	5/12	1/4

8. Consider a random variable X with $\mu_X = 3$ and $\sigma_X^2 = 0.25$. Find:
   a. μ_{3+6X}
   b. σ_{3+6X}

9. Consider two discrete independent, random variables X and Y with $\mu_X = 3, \sigma_X^2 = 1, \mu_Y = 5,$ $\sigma_Y^2 = 1.3$. Find μ_{X-Y} and σ_{X-Y}.

10. Consider the following two probability distributions for independent discrete random variable X and Y:

X	2	3	4
P(X)	0.3	0.5	?

Y	3	4	5	6
P(Y)	?	0.1	?	0.4

If P(X = 4 and Y = 3) = 0.03, what is P(Y = 5)?

11. Consider a random variable X with the following probability distribution:

X	20	21	22	23	24
P(X)	0.2	0.3	0.2	0.1	0.2

   a. Find P(X ≤ 22)
   b. Find P(X > 21)
   c. Find P(21 ≤ X < 24)
   d. Find P(X ≤ 21 or X > 23)

12. In the casino game of roulette, a ball is rolled around the rim of a circular bowl while a wheel containing 38 slots into which the ball can drop is spun in the opposite direction from the rolling ball; 18 of the slots are red, 18 are black, and 2 are green. A player bets a set amount, say $1, and win $1 (and keeps her $1 bet) if the ball falls into the color slot the player has wagered on. Assume a player decides to bet that the ball will fall into one of the red slots.
   a. What is the probability that the player will win?
   b. What is the expected return on a single bet of $1 on red?

13. A factory manufacturing tennis balls determines that the probability that a single can of three balls will contain at least one defective ball is 0.025. What is the probability that a case of 48 cans will contain at least two cans with a defective ball?

14. Suppose you had gobs of time on your hands and decided to flip a fair coin 1,000,000 times and note whether each flip was a head or a tail. Let X be the count of heads. What is the probability that there are at least 1000 more heads than tails? (Note: this is a binomial distribution but your calculator will not be able to do the binomial computation because the numbers are too large for it).

15. At a school better known for football than academics (a school its football team can be proud of), it is known that only 20% of the scholarship athletes graduate within 5 years. The school is able to give 55 scholarships for football. What are the expected mean and standard deviation of the number of graduates for a group of 55 scholarship athletes?

16. Approximately 10% of the population of the United States is known to have blood type B. If this is correct, what is the probability that between 11% and 15%, inclusive, of a random sample of 500 adults will have type B blood?

17. A brake inspection station reports that 15% of all cars tested have brakes in need of replacement pads. For a sample of 20 cars that come to the inspection station,
    a. What is the probability that exactly 3 have defective breaks?
    b. What is the mean and standard deviation of the number of cars that need replacement pads?

18. The probability that a person recovers from a particular type of cancer operation is 0.7. Suppose 8 people have the operation. What is the probability that
    a. Exactly 5 recover?
    b. They all recover?
    c. At least one recovers?

19. After the *Challenger* disaster of 1986, it was discovered that the explosion was caused by defective O-rings. The probability that a single O-ring was defective and would fail (with catastrophic consequences) was 0.003 and there were 12 of them (6 outer and 6 inner). What was the probability that at least one of the O-rings would fail (as it actually did)?

20. Your favorite cereal has a little prize in each box. There are 5 such prizes. Each box is equally likely to contain any one of the prizes. So far, you have been able to collect 2 of the prizes. What is:
    a. The probability that you will get the third different prize on the next box you buy?
    b. The probability that it will take three more boxes to get the next prize?
    c. The average number of boxes you will have to buy before getting the third prize?

SOLUTIONS

MULTIPLE CHOICE.

1) C

The correct answer is (c). The expected value is $(-1)(0.6) + (1)(0.25) + (2)(0.15) = -0.05$.

2) A

The correct answer is (a).

$$\mu_X = (60)(0.4) = 24, \sigma_X = \sqrt{60(0.4)(0.6)} = \sqrt{14.4} = 3.79.$$

3) C

The correct answer is (c). Although you probably wouldn't need to use a normal approximation to the binomial for small sample sizes, there is no reason (except perhaps accuracy) that you couldn't.

4) D

The correct answer is (d). Because the problem stated "at least 10," we must include the term where $x = 10$. If the problem has said "more than 10," the correct answer would have been (b) or (c) (they are equivalent). The answer could also have been given as

$$\binom{50}{10}(0.083)^{10}(0.917)^{40} + \binom{50}{11}(0.083)^{11}(0.917)^{39} + \ldots + \binom{50}{50}(0.083)^{50}(0.917)^{0}.$$

5) B

The correct answer is (b). This is a characteristic of a binomial experiment. The analogous characteristic for a geometric experiment is that there is a random variable X that is the number of trials needed to achieve the first success.

6) C

The correct answer is (c). This is actually a binomial situation. If X is the count of students "in favor," then X has B(200, 0.70). Thus, $P(X \geq 150) = P(X = 150) + P(X = 151) + \ldots + P(X = 200)$. Using the TI-83/84, the exact binomial answer equals `1-binomcdf (200,0.7.0,149)=0.0695`. None of the listed choices shows a sum of several binomial expressions, so we assume this is to be done as a normal approximation. We note that B(200, 0.7) can be approximated by $N(200(0.7), \sqrt{200(0.7)(0.3)}) = N(140, 6.4807)$. A normal approximation is OK since 200(0.7) and 200(0.3) are both much greater than 10. Since 75% of 200 is 150, we have $P(X \geq 150) = P\left(z \geq \frac{150-140}{6.487} = 1.543\right) = 0.614.$

FREE RESPONSE

7)

$$\mu_x = 2\left(\frac{1}{3}\right) + 3\left(\frac{5}{12}\right) + 4\left(\frac{1}{4}\right) = \frac{35}{12} \approx 2.92$$

$$\sigma_X = \sqrt{\left(2-\frac{35}{12}\right)^2\left(\frac{1}{3}\right) + \left(3-\frac{35}{12}\right)^2\left(\frac{5}{12}\right) + \left(4-\frac{35}{12}\right)^2\left(\frac{1}{4}\right)} = 0.759.$$

8)

(a) $\mu_{3+6X} = 3 + 6\mu_X = 3 + 6(3) = 21.$

(b) Because $\sigma^2_{a+bx} = b^2\sigma^2, \sigma^2_{3+6x} = 6^2\sigma^2_x = 36(0.25) = 9.$

Thus,

$$\sigma_{3+6x} = \sqrt{\sigma^2_{3+6X}} = \sqrt{9} = 3.$$

9)

$\mu_{X-Y} = \mu_X - \mu_Y = 3 - 5 = -2$.

Since X and Y are independent, we have $\sigma_{X-Y} = \sqrt{\sigma^2_X + \sigma^2_Y} = \sqrt{1+1.3} = 1.52$. Note that the variances add even though we are subtracting one random variable from another.

10)

Since $\sum P(X) = 1$, we have $P(X = 4) = 1 - P(X = 2) - P(X = 3) = 1 - 0.3 - 0.5 = 0.2$. Thus, filling in the table for X, we have

X	2	3	4
***P*(X)**	0.3	0.5	**0.2**

Since X and Y are independent, $P(X = 4 \text{ and } Y = 3) = P(X = 4) \bullet P(X = 3)$. We are given that $P(X = 4 \text{ and } Y = 3) = 0.03$. Thus, $P(X = 4) \bullet P(Y = 3) = 0.03$. Since we now know that $P(X = 4) = 0.2$, we have $(0.2).\ P(Y = 3) = 0.03$, which gives us $P(Y = 3) = \dfrac{0.03}{0.2} = 0.15$.

Now, since $\sum P(Y) = 1$, we have $P(Y = 5) = 1 - P(Y = 3) - P(Y = 4) - P(Y = 6) = 1 - 0.15 - 0.1 - 0.4 = 0.35$.

11)

(a) $P(x \le 22) = P(x = 20) + P(x = 21) + P(x = 22) = 0.2 + 0.3 + 0.2 = 0.7.$

(b) $P(x > 21) = P(x = 22) + P(x = 23) + P(x = 24) = 0.2 + 0.1 + 0.2 = 0.5.$

(c) $P(21 \le x < 24) = P(x = 21) + P(x = 22) + P(x = 23) = 0.3 + 0.2 + 0.1 = 0.6.$

(d) $P(x \le 21 \text{ or } x > 23) = P(x = 20) + P(x = 21) + P(x = 24) = 0.2 + 0.3 + 0.2 = 0.7.$

12)

(a) 18 of the 38 slots are winners, so $P(\text{win if bet on red}) = \frac{18}{38} = 0.474$.

(b) The probability distribution for this game is

Outcome	Win	Lose
X	1	-1
$P(X)$	$18/38$	$20/38$

$$E(X) = \mu_X = \left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right) = -0.052 \text{ or } -5.2¢.$$

The player will lose 5.2¢, on average, for each dollar bet.

13)

If X is the count of cans with at least one defective ball, then X has $B(48, 0.025)$.

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) =$$
$$1 - \binom{48}{0}(0.025)^0(0.975)^{48} - \binom{48}{1}(0.025)^1(0.975)^{47} = 0.338.$$

On the TI-83/84, the solution is given by `1-binomcdf(48,0.025,1)`.

14)

Since the `binomcdf` function can't be used due to calculator overflow, we will use a normal approximation to the binomial. Let X = the count of heads. Then $\mu_X = (1{,}000{,}000)(0.5) = 500{,}000$ (assuming a fair coin) and $\sigma_X = \sqrt{(1{,}000{,}000)(0.5)(0.5)} = 500$. Certainly both np and $n(1-p)$ are greater than 5 (waaaaaay larger!), so the conditions needed to use a normal approximation are present. If we are to have at least 1,000 more heads than tails, then there must be at least 500,500 heads (and, of course, no more than 499,500 tails). Thus, P(there are at least 1,000 more heads than tails) $= P(X) \geq 500500 = P\left(z \geq \frac{500{,}500 - 500{,}000}{500} = 1\right) = 0.1587$.

15)

If X is the count of scholarship athletes that graduate from any sample of 55 players, then X has $B(55, 0.20)$. $\mu_X = 55(0.20) = 11$ and $\sigma_X = \sqrt{55(0.20)(0.80)} = 2.97$.

16)

There are three different ways to do this problem: exact binomial, using proportions, or using a normal approximation to the binomial. The last two are essentially the same.

(i) **Exact binomial.** Let X be the count of persons in the sample that have blood type B. Then X has $B(500, 0.10)$. Also, 11% of 500 is 55 and 15% of 500 is 75. Hence, $P(55 \le X \le 75) = P(X \le 75) - P(X \le 54) \approx$ `binomcdf(500,0.10,75)-binomcdf(500,0.10,54)=0.2475.`

(ii) **Proportions.** We note that $\mu_X = np = 500(0.1) = 50$ and $n(1-p) = 500(0.9) = 90$, so we are OK to use a normal approximation. Also, $\mu_{\hat{p}} = p = 0.10$ and $\sigma_{\hat{p}} \approx \sqrt{\frac{(0.1)(0.9)}{500}} \approx 0.0134$. $P(0.11 \le \hat{p} \le 0.15) \approx P\left(\frac{0.11-0.10}{0.0134} \le z \le \frac{0.15-0.10}{0.0134}\right) = P(0.7463 \le z \le 3.731) = 0.2276$. On the TI 83/84: `normalcdf(0.7463,3.731)`.

(iii) **Normal approximation to the binomial.** The conditions for doing a normal approximation were established in part (ii). Also, $\mu_X = 500(0.1) = 50$ and $\sigma_X = \sqrt{500(0.1)(0.9)} = 6.7082$. $P(55 \le X \le 75) = P\left(\frac{55-50}{6.7082} \le z \le \frac{75-50}{6.7082}\right) = P(0.7454 \le z \le 3.7268) = 0.2279$.

17)

If X is the count of cars with defective pads, then X has $B(20, 0.15)$.

(a) $P(X = 3)\binom{20}{3}(0.15)^3(0.85)^{17} = 0.243$. On the TI-83/84, the solution is given by `binompdf(20,0.15,3)`.

(b) $\mu_X = np = 20(0.3) = 6$, $\sigma_X = \sqrt{np(1-p)} = \sqrt{20(0.3)(1-0.3)} = 2.049$.

18)

If X is the number that recover, then X has $B(8, 0.7)$.

(a) $P(X = 5) = \binom{8}{5}(0.7)^5(0.3)^3 = 0.254$. On the TI-83/84, the solution is given by `binompdf(8,0.7,5)`.

(b) $P(X = 8) = \binom{8}{8}(0.7)^8(0.3)^0 = 0.058$. On the TI-83/84, the solution is given by `binompdf(8,0.7,8)`.

(c) $P(X \ge 1) = 1 - P(X = 0) = 1 - \binom{8}{0}(0.7)^0(0.3)^8 = 0.999$. On the TI-83/84, the solution is given by `1-binompdf(8,0.7,0)`.

19)

If X is the count of O-rings that failed, then X has $B(12, 0.003)$.

$$P(\text{at least one fails}) = P(X=1)+P(X=2)+\cdots+P(X=12)$$
$$=1-P(X=0)=1-\binom{12}{0}(0.003)^0(0.997)^{12}=0.035.$$

On the TI-83/84, the solution is given by `1-binompdf(12,0.003,0)`.
The clear message here is that even though the probability of any one failure seems remote (0.003), the probability of at least one failure (3.5%) is large enough to be worrisome.

20)

Because you already have 2 of the 5 prizes, the probability that the next box contains a prize you don't have is 3/5 = 0.6. If n is the number of trials until the first success, then $P(X = n) = (0.6).\ (0.4)^{n-1}$.

(a) $P(X = 1) = (0.6)(0.4)^{1-1} = (0.6)(1) = 0.6$. On the TI-83/84 calculator, the answer can be found by `geometpdf(0.6,1)`.

(b) $P(X = 3) = (0.6)(0.4)^{3-2} = 0.096$. On the calculator: `geometpdf(0.6,3)`.

(c) The average number of boxes you will have to buy before getting the third prize is $\frac{1}{0.6} = 1.67$.