Chapter 7 Review

MULTIPLE CHOICE.

1) You form a distribution of the means of all samples of size 9 drawn from an infinite population that is skewed to the left (like the scores on an easy Stats quiz!). The population from which the samples are drawn has a mean of 50 and a standard deviation of 12. Which one of the following statements is true of this distribution?
   a) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$, the sampling distribution is skewed somewhat to the left.
   b) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 4$, the sampling distribution is skewed somewhat to the left.
   c) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$, the sampling distribution is approximately normal.
   d) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 4$, the sampling distribution is approximately normal.
   e) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 4$, the sample size is too small to make any statements about the shape of the sampling distribution.

2) In a large population, 55% of the people get a physical examination at least once every two years. A SRS of 100 people are interviewed and the sample proportion is computed. The mean and standard deviation of the sampling distribution of the sample proportion are
   a) 55, 4.97
   b) 0.55, 0.002
   c) 55, 2
   d) 0.55, 0.0497
   e) The standard deviation cannot be determined from the given information.

3) Which of the following best describes the sampling distribution of a sample mean?
   a) It is the distribution of all possible sample means of a given size.
   b) It is the particular distribution in which $\mu_{\bar{x}} = \mu \textit{ and } \sigma_{\bar{x}} = \sigma$.
   c) It is a graphical representation of the means of all possible samples.
   d) It is the distribution of all possible sample means from a given population.
   e) It is the probability distribution for each possible sample size.

FREE RESPONSE

4) A population is highly skewed to the left. Describe the shape of the sampling distribution of $\bar{x}$ drawn from this population if the sample size is (a) 3 or (b) 30.

5) Consider a population consisting of the numbers 2, 4, 5, and 7. List all possible samples of size two from this population and compute the mean and standard deviation of the sampling distribution of $\bar{x}$. Compare this with the values obtained by relevant formulas for the sampling distribution of $\bar{x}$. Note that the sample size is large relative to the population – this may affect how you compute $\sigma_{\bar{x}}$ by formula.

6) Which of the following is/are true of the central limit theorem? (More than one answer might be true.)
   I. $\mu_{\bar{x}} = \mu$
   II. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ (if N ≥ 10n)
   III. The sampling distribution of a sample mean will be approximately normally distributed for sufficiently large samples, regardless of the shape of the original population.
   IV. The sampling distribution of a sample mean will be normally distributed if the population from which the samples are drawn is normal.

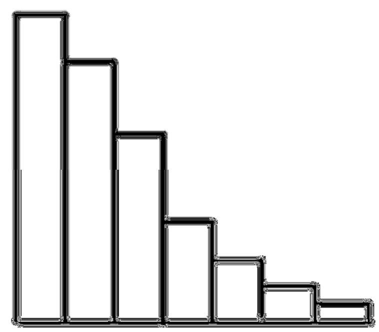
7) A tire manufacturer claims that his tires will last 40,000 miles with a standard deviation of 5000 miles.
   a) Assuming that the claim is true, describe the sampling distribution of the mean lifetime of a random sample of 160 tires. Remember that "describe" means discuss center, spread, and shape.
   b) What is the probability that the mean life time of the sample of 160 tires will be less than 39,000 miles? Interpret the probability in terms of the truth of the manufacturer's claim.

8) Crabs off the coast of Northern California have a mean weight of 2 lbs with a standard deviation of 5 oz. A large trap captures 35 crabs.
   a) Describe the sampling distribution for the average weight of a random sample of 35 crabs taken from this population.
   b) What would the mean weight of a sample of 35 crabs have to be in order to be in the top 10% of all such samples?

9) A certain type of light bulb is advertised to have an average life of 1200 hours. If, in fact, light bulbs of this type only average 1185 hours with a standard deviation of 80 hours, what is the probability that a sample of 100 bulbs will have an average life of at least 1200 hours?

10) Your task is to explain to your friend Gretchen, who know virtually nothing (and cares even less) about statistics, just what the sampling distribution of the mean is. Explain the idea of a sampling distribution in such a way that even Gretchen, if she pays attention, will understand.

11) Consider the distribution shown at the right. Describe the shape of the sampling distribution of $\bar{x}$ for samples of size n if
   a) n = 3
   b) n = 40



12) Opinion polls in 2002 showed that about 70% of the population had a favorable opinion of President Bush. The same year, a simple random sample of 600 adults living in the San Francisco Bay Area showed only 65% had a favorable opinion of President Bush. What is the probability of getting a rating of 65% or less in a random sample of this size if the true proportion in the population was 0.70?

SOLUTIONS

MULTIPLE CHOICE.

1)

The answer is (b).

$$\mu_{\bar{x}} = \mu,\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

For small samples, the shape of the sampling distribution of $\bar{x}$ will resemble the shape of the sampling distribution of the original population. The shape of the sampling distribution of $\bar{x}$ is approximately normal for n sufficiently large.

2)

The correct answer is (d). $\mu_{\hat{p}} = p = 0.55, \sigma_{\hat{p}} = \sqrt{\frac{(0.55)(0.45)}{100}} = 0.0497.$

3)

The correct answer is (a).

FREE RESPONSE

4)

We know that the sampling distribution of $\bar{x}$ will be similar to the shape of the original population for small n and approximately normal for large n (that's the central limit theorem). Hence,

(a) if $n = 3$, the sampling distribution would probably be somewhat skewed to the left.
(b) if $n = 30$, the sampling distribution would be approximately normal.

Remember that using $n \geq 30$ as a rule of thumb for deciding whether to assume normality is for a sampling distribution just that: a rule of thumb. This is probably a bit conservative. Unless the original population differs markedly from mound shaped and symmetric, we would expect to see the sampling distribution of $\bar{x}$ be approximately normal for considerably smaller values of n.

5)

Putting the numbers 2, 4, 5, and 7 into a list in a calculator and doing `1-Var Stats`, we find $\mu = 4.5$ and $\sigma = 1.802775638$. The set of all samples of size 2 is {(2,4), (2,5), (2,7), (4,5), (4,7), (5,7)} and the means of these samples are {3, 3.5, 4.5, 4.5, 5.5, 6}. Putting the means into a list and doing `1-Var Stats` to find $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$, we get $\mu_{\bar{x}} = 4.5$ (which agrees with the formula) and $\sigma_{\bar{x}} = 1.040833$ (which does not agree with $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.802775638}{\sqrt{2}} = 1.27475878$). Since the sample is large compared with the population (that is, the population isn't at least 20 times as large as the sample), we use $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{1.802775638}{\sqrt{2}}\sqrt{\frac{4-2}{4-1}} = 1.040833$, which does agree with the computed value.

6)

All four of these statement are true. However, only III is a statement of the central limit theorem. The others are true of sampling distributions in general.

7)

$\mu_{\bar{x}} = 40{,}000$ miles and $\sigma_{\bar{x}} = \dfrac{5000}{\sqrt{160}} = 395.28$ miles.

(a) With $n = 160$, the sampling distribution of $\bar{x}$ will be approximately normally distributed with mean equal to 40,000 miles and standard deviation 395.28 miles.

(b) $P(\bar{x} < 39{,}000) = P\left(z < \dfrac{39{,}000 - 40{,}000}{395.28} = -2.53\right) = 0.006.$

If the manufacturer is correct, there is only about a 0.6% chance of getting an average this low or lower. That makes it unlikely to be just a chance occurrence and we should have some doubts about the manufacturer's claim.

8)

$\mu_{\bar{x}} = 2\text{ lbs} = 32\text{ oz}$ and $\sigma_{\bar{x}} = \dfrac{5}{\sqrt{35}} = 0.845\text{ oz.}$

(a) With samples of size 35, the central limit theorem tells us that the sampling distribution of $\bar{x}$ is approximately normal. The mean is 32 oz. and standard deviation is 0.845 oz.

(b) In order for $\bar{x}$ to be in the top 10% of samples, it would have to be at the 90th percentile, which tells us that its z-score is 1.28 [that's InvNorm(0.9) on your calculator]. Hence,

$$z_{\bar{x}} = 1.28 = \frac{\bar{x} - 32}{0.845}.$$

Solving, we have $\bar{x} = 33.08$ oz. A crab would have to weigh at least 33.08 oz, or about 2 lb 1 oz, to be in the top 10% of samples of this size.

9)

$\mu_{\bar{x}} = 1185$ hours, and $\sigma_{\bar{x}} = \dfrac{80}{\sqrt{100}} = 8$ hours.

$$P(\bar{x} \geq 1200) = P\left(z \geq \frac{1200 - 1185}{8} = 1.875\right) = 0.03.$$

10)

The first thing Gretchen needs to understand is that a distribution is just the set of all possible values of some variable. For example the distribution of SAT scores for the current senior class is just the values of all the SAT scores. We can draw samples from that population if, say, we want to estimate the average SAT score for the senior class but don't have the time or money to get all the data. Suppose we draw samples of size n and compute $\bar{x}$ for each sample. Imagine drawing ALL possible samples of size n from the original distribution (that was the set of SAT scores for everybody in the senior class). Now consider the distribution (all the values) of means for those samples. That is what we call the sampling distribution of $\bar{x}$ (the short version: the sampling distribution of $\bar{x}$ is the set of all possible values of $\bar{x}$ computed from samples of size n.)

11)

The distribution is skewed to the right.

(a) If $n = 3$, the sampling distribution of $\bar{x}$ will have some right skewness, but will be more mound shaped than the parent population.

(b) If $n = 40$, the central limit theorem tells us that the sampling distribution of $\bar{x}$ will be approximately normal.

12)

If $p = 0.70$, then $\mu_{\hat{p}} = 0.70$ and $\sigma_{\hat{p}} = \sqrt{\dfrac{0.70(1-0.70)}{600}} = 0.019$. Thus, $P(\hat{p} \le 0.65) =$

$P\left(z \le \dfrac{0.65-0.70}{0.019} = -2.63\right) = 0.004$. Since there is a very small probability of getting a sample proportion as small as 0.65 if the true proportion is really 0.70, it appears that the San Francisco Bay Area may not be representative of the United States as a whole (that is, it is unlikely that we would have obtained a value as small as 0.65 if the true value were 0.70).