Section 8.1 Notes
Estimating a Proportion with Confidence

A confidence interval for the proportion of successes \( p \) in the population given by the formula: ____________________________

Margin of Error: half the width of the confidence interval: _______________________________

Reasonably accurate when three conditions are met:
- The sample was a ____________________________ from a binomial population.
- Both \( np \) and \( n(1-p) \) are at least ___________
- The size of the population is at least ________________ the size of the sample

### Common Confidence Intervals

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>( \alpha )</th>
<th>Critical Value, ( z^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>95%</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>99%</td>
<td>0.01</td>
<td></td>
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</tbody>
</table>

**Interpretation of Confidence Interval:**

Example: 95% confidence interval
- You are 95% confident if you could examine each unit in the population, the proportion of successes, \( p \), in this population would be in this confidence interval.
- If you were able to repeat this process 100 times and construct the 100 resulting confidence intervals, you would expect 95 of them to contain the population proportion of successes, \( p \). In other words, 95% of the time the process results in a confidence interval that captures the true value of \( p \).

**Example:**
In the first two months of a recent year, 94 car occupants were killed by air bags, and 61 of them were "improperly buckled".
1. Construct a 95% confidence interval estimate of the percentage of car occupants who were killed by air bags while being improperly buckled. Interpret the interval in context.

2. Based on the results, is it safe to say that the majority of car occupants killed by air bags were improperly belted?

**Sample size for estimating proportion \( p \):**

Note: If \( \hat{p} \) is unknown use \( p = \frac{1}{2} \)

**Example:**
3. The US Crime Commission wants to estimate the proportion of crimes in which firearms were used to within 0.02 with 90% confidence. Data from previous years show that the proportion of crimes in which firearms is used is about 60%. Find the sample size they will need.
Basic Definitions:

- In Statistics, a _____________________________ is a claim or statement about a property of a population.
- A ___________________________________ (or test of significance) is a standard procedure for testing a claim about a property of a population.
- A sample proportion is said to be _____________________________ if it isn’t a reasonably likely outcome when the proposed standard is true.

**COMPONENTS OF A FORMAL HYPOTHESIS TEST**

#1: Give the name of the test and check the conditions for its use.

- The sample was a _____________________________________ from a binomial population.
- Both np and n(1-p) are at least ____________
- The size of the population is at least _______________________ the size of the sample.

#2: State the hypotheses, defining any symbols.

- The _________________________________________ (denoted __________) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is ________________________ some claimed value.
  - We test the null hypothesis directly: we assume that it is true and reach a conclusion to either ________________ or ________________________________
- The _________________________________________ (denoted _________) is the statement that the parameter has a value that somehow differs from the null hypothesis (usually involves an inequality)

- A few notes about H₀ and H₁:
  - We conduct the hypothesis test by assuming the parameter is ________________ some specific value so that we can work with a single distribution having a _________________________.
  - If you are conducting a study and want to use a hypothesis test to ________________ your claim, the claim must be worded so that it becomes the __________________ hypothesis. You can never support a claim that some parameter is __________________ some specified value.
  - Note that the original statement could become the ____________________________, it could become the ____________________________, or it might not correspond exactly to either.

#3: Compute the test statistic, z, and find the critical value, z*, and the P-value. Include a sketch that illustrates the situation.

- The ___________________________________ is a value used in making a decision about the null hypothesis, and it is found by converting the sample statistic to a score with the assumption that the null hypothesis is true.
- Formula:

- Critical Region, Significance Level, Critical Value, and P-Value
  - The ___________________________________ (or rejection region) is the set of all values of the test statistic that causes us to reject the null hypothesis.
  - The ___________________________________ (denoted by _____) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true.
    - If the test statistic falls in the critical region, we ________________ the null hypothesis, so _________ is the probability of making the mistake of rejecting the null hypothesis when it is true.
A _________________________________ is any value that separates the critical region from the values of the test statistic that do not lead to rejection of the null hypothesis.

The ________________________________ is the probability of getting a value of the test statistic that is at least as extreme as the one representing the sample data, assuming the null hypothesis is true.

#4: Write a conclusion. (Two parts)
- Determine whether to reject or fail to reject the null hypothesis, linking the reason to the P-value or to the critical values.
- Tell what your conclusion means in the context of the situation.

Decisions and Conclusions
The decision to reject or fail to reject the null hypothesis can be made by using any of the following methods:

- ____________________________
  o ____________________________ if the test statistic falls within the critical region.
  o ____________________________ if the test statistic does not fall within the critical region.
- ____________________________
  o ____________________________ if the p-value ≤ α (where α is the significance level, such as 0.05)
  o ____________________________ if the p-value > α

<table>
<thead>
<tr>
<th>AP Exam-Hypothesis Test Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) <strong>Hypothesis</strong></td>
</tr>
<tr>
<td>• State the Null Hypothesis</td>
</tr>
<tr>
<td>• State Alternative Hypothesis (with proper tails)</td>
</tr>
<tr>
<td>• Use correct notation (define if non-standard)</td>
</tr>
<tr>
<td>2) <strong>Test</strong></td>
</tr>
<tr>
<td>• Check assumptions and conditions (not just list)</td>
</tr>
<tr>
<td>• Specify the model</td>
</tr>
<tr>
<td>• Name the test</td>
</tr>
<tr>
<td>3) <strong>Mechanics</strong></td>
</tr>
<tr>
<td>• Show work (statistics, values subbed into formula, shaded sketch of model, etc.)</td>
</tr>
<tr>
<td>• Report test statistic (z, t, x², df)</td>
</tr>
<tr>
<td>• Report P-value</td>
</tr>
<tr>
<td>4) <strong>Conclusions</strong></td>
</tr>
<tr>
<td>• State the decision (Reject, Fail to Reject)</td>
</tr>
<tr>
<td>• With linkage to the P-value (&quot;because the P-value is so low...&quot; or something like that)</td>
</tr>
<tr>
<td>• Interpret the result in context</td>
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</tbody>
</table>

Example:
Ships arriving in the US ports are inspected by Customs officials by contaminated cargo. Assume for a certain port, 20% of the ships arriving in the previous year contained cargo that was contaminated. A random selection of 50 ships in the current year included 5 that had contaminated cargo. Does the data suggest that the proportion of ships arriving in the port with contaminated cargoes has decreased in the current year? Use α = 0.01.
Type I and Type II Errors

<table>
<thead>
<tr>
<th>We decide to reject the null hypothesis</th>
<th>The null hypothesis is true</th>
<th>Type I Error (rejecting a true null hypothesis)</th>
<th>Correct Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>We fail to reject the null hypothesis</td>
<td>Correct Decision</td>
<td>Type II Error (failing to reject a false null hypothesis), $\beta$</td>
<td></td>
</tr>
</tbody>
</table>

Example: Identify the type I and type II error that corresponds to the given hypothesis.
The proportion of LHS students that are seniors is 0.27.

Type I Error:
- To decrease the probability of a Type I Error, make ____________________. Changing the sample size has ________________ on the probability of a Type I Error.
- If the null hypothesis is false, you can't make a Type I Error.

Type II Error:
- To decrease the probability of a Type II Error, take a ____________________ or make ____________________
- If the null hypothesis is true, you can't make a Type II Error.

Power:
- Power: the probability of rejecting a null hypothesis.
- When the null hypothesis is ________________, you want to reject it and therefore you want the power to be ___________
- To increase power, you can either take a ____________________ or make ____________________.
CI for Difference of Two Proportions

Conditions that must be met:

- The two samples are taken _________ and _________ from two populations.
- Each population is at least _________ as large as its sample size
- __________________, ________________, ________________, ________________ are all at least ______.

CI for the difference $p_1 - p_2$:

$$\text{statistic} \pm \text{critical value} \cdot \text{standard deviation of the statistic}$$

Example:
Suppose the Cartoon Network conducts a nationwide survey to assess viewer attitudes toward Superman. Using a simple random sample, they select 400 boys and 300 girls to participate in the study. Forty percent of the boys say that Superman is their favorite character, compared to thirty percent of the girls. What is the 90% confidence interval for the true difference in attitudes toward Superman?
Section 8.4 Notes
Significance Test for Difference of Two Proportions

Conditions that must be met:

- The two samples are taken _________________________ and ____________________________ from two populations
- Each population is at least ______________________ as large as its sample size
- __________________, __________________, ___________________, __________________ are at least _________

$H_0$ and $H_a$:

Forms of $H_0$: _______________________________________

Forms of $H_a$: _______________________________________

Test Statistic:

$\hat{p} = \frac{\text{total number of successes in both samples}}{n_1 + n_2}$

Example:
A survey was conducted of students from Cincinnati Public Schools system to determine if the incidence of hungry children was consistent in two schools located in lower-income areas. A random sample of 80 elementary students from school A found that 23% did not have breakfast before coming to school. A random sample of 180 elementary students from school B found that 7% did not have breakfast before coming to school.
EXPERIMENTS

Conditions (for both CI and HT):

- The two treatments are ______________________________ to the population of available experimental units.
- __________, __________, __________, __________ are all greater than __________

Example:
In 1954, the largest medical experiment of all times was carried out to test whether the newly developed Salk vaccine was effective in preventing polio. This study incorporated all three characteristics of an experiment: use of a control group of children who received a placebo injection, random assignment of children to either the placebo injection group or the Salk vaccine injection group, and assignment of each treatment to several hundred thousand children. Of the 200,745 children who received the Salk vaccine, 82 were diagnosed with polio. Of the 201,229 children who received the placebo, 162 were diagnosed with polio.

OBSERVATIONAL STUDY

- Treatments are NOT randomly assigned
- DO NOT allow you to make __________________________________ conclusions or ________________________________ in a rigorous way, but they can provide evidence of a possible association.
  - Why? _________________________________________________________________

Example:
Researchers wanted to study the effects of exercising on dementia. They followed a group of people age 65 or older from 1994 to 2003. Among the 1185 free of dementia at the end of this time, 77% reported exercising three or more times a week. Among the 158 who showed signs of dementia at the end of the period, 67% reported exercising three or more times per week.