AP Stats
Central Limit Theorem

Name: ______________________

1. An insurance company has found that repair claims have a mean of \$525 with standard deviation of \$375. Because most of the claims are for minor repairs and a few are for very extensive work, the distribution is skewed to the right.

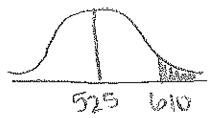
   A. A simple random sample of 60 repairs is recorded. What is the mean, standard deviation, and shape of the distribution of $\bar{x}$.

   mean = 525

   stan. dev. = $375/\sqrt{60} = 48.412$

   normal dist.

   B. What is the probability that the mean repair cost for these 60 claims is greater than \$610?



   $z = \dfrac{610-525}{375/\sqrt{60}} = 1.76$ $\quad 1-.9608 = \boxed{.0392}$

   C. What is the probability that the mean repair cost for these 60 claims is between \$500 and \$600?

   $z = \dfrac{600-525}{375/\sqrt{60}} = 1.55$ $\quad .9394$

   $z = \dfrac{500-525}{375/\sqrt{60}} = -.52$ $\quad -.3015$

   $\boxed{.6379}$

   D. Ninety percent of all sample means of size 60 will be in the interval $\$525 \pm M$. Find the value of M.

   $X = 525 \pm \underbrace{1.645\left(\dfrac{375}{\sqrt{60}}\right)}_{=M}$

   $\boxed{M = 79.638}$

   $\pm 1.645 = \dfrac{X-525}{\frac{375}{\sqrt{60}}}$

   $\pm 1.645(375/\sqrt{60}) = X - 525$
   $+525 \qquad +525$

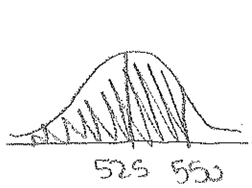
   E. How would your answer C change if the sample size is increased to 100? Explain.

   $z = \dfrac{600-525}{375/\sqrt{100}} = 2$ $\quad .9772$

   $z = \dfrac{500-525}{375/\sqrt{100}} = -.67$ $\quad -.2514$

   $\boxed{.7258}$

   F. The insurance company wants to choose a sample size for which $P(\bar{x} \le 550) \ge 90\%$. Find the smallest sample size needed for this to be true.



   $1.28 = \dfrac{550-525}{\frac{375}{\sqrt{n}}}$

   $\dfrac{1.28\left(\frac{375}{\sqrt{n}}\right)}{1.28} = \dfrac{25}{1.28}$

   $\dfrac{375}{\sqrt{n}} = 19.531$

   $375 = 19.531\sqrt{n}$

   $19.2^2 = \sqrt{n}^2$

   $n = 368.64$

   $\boxed{369}$

2. The Mars Company makes M&M's and advertises that 30% of all plain M&M'2 are brown. A SRS of 125 plain M&M's is drawn.

A. Find the mean and standard deviation of $\hat{p}$, the proportion of the sample that are brown.

$\mu_{\hat{p}} = .30$

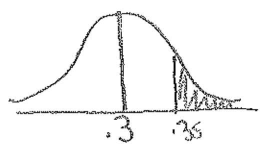
$\sigma_{\hat{p}} = \sqrt{\frac{(.3)(.7)}{125}} = .041$

B. Show that the conditions are satisfied that will ensure that the distribution of $\hat{p}$ is approximately normal.

$.3(125) = 37.5 > 10$

$.7(125) = 87.5 > 10$

C. Find the probability that the proportion of brown M&M's is greater than 35%.



$z = \frac{.35 - .3}{.041} = 1.22$

$1 - .8888 = \boxed{.1112}$

D. Find the probability that the proportion of brown M&M's is between 25% and 35%.

$z = \frac{.25 - .3}{.041} = -1.22$

$z = \frac{.35 - .3}{.041} = 1.22$

$.8888 - .1112$

$\boxed{.7776}$

E. If the sample is increased to 200, how will the answer to C change? Explain.

$z = \frac{.35 - .3}{\sqrt{\frac{(.3)(.7)}{200}}} = 1.54 \quad 1 - .9382 \quad \boxed{.0618}$

F. If the sample is increased to 200, how will the answer to D change? Explain.

$.9382 - .0618 = \boxed{.8764}$

G. How large must the sample size be so that 90% of all sample proportion will be within 2% of the population proportion p = 30%.

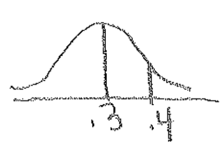
$\left(1.645\sqrt{\frac{(.3)(.7)}{n}}\right)^2 = .02^2$

$\frac{2.706(.3)(.7)}{n} = .02^2$

$1.645 = \frac{.32 - .3}{\sqrt{\frac{(.3)(.7)}{n}}}$

$n = 1420.06 \quad \boxed{1421}$

H. A sample of 125 peanut M&M's is drawn and it is found that 50 of the 125 are brown. If peanut M&M's are distributed with the same proportions as the plain M&M's, what is the probability of obtaining a sample proportion as large or larger than this one. What can be said about the proportion of brown peanut M&M's.



$\frac{.4 - .3}{.041} = 2.44$

$1 - .9927$

$\boxed{.0073}$

It is not likely plain & peanut M&Ms have same dist