Section 8.2 Notes
Testing a Proportion

Basic Definitions:

- In statistics, a ___hypothesis___ is a claim or statement about a property of a population.

- A ___hypothesis test___ (or test of significance) is a standard procedure for testing a claim about a property of a population.

- A sample proportion is said to be ___statistically significant___ if it isn't a reasonably likely outcome when the proposed standard is true.

COMPONENTS OF A FORMAL HYPOTHESIS TEST

1) Give the name of the test and check the conditions for its use.

For proportions, three conditions must be met:

- The sample was a ___Simple Random Sample___ from a binomial population.

- Both np and n(1 - p) are at least ___10___

- The size of the population is at least ___10 times___ the size of the sample.

2) State the hypotheses, defining any symbols.

Null and Alternative Hypotheses

The ___null hypothesis___ (denoted ___H_0___) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is ___equal to___ some claimed value.

We test the null hypothesis directly: we assume that it is true and reach a conclusion to either ___reject H_0___ or ___fail to reject H_0___.

The ___alternative hypothesis___ (denoted by ___H_1 or H_a___) is the statement that the parameter has a value that somehow differs from the null hypothesis (usually involves and inequality).

A few notes about H_0 and H_a:

- We conduct the hypothesis test by assuming the parameter is ___equal to___ some specified value so that we can work with a single distribution having a ___specific value___.
- If you are conducting a study and want to use a hypothesis test to ___support___ your claim, the claim must be worded so that it becomes the ___alternative___ hypothesis. You can never support a claim that some parameter is ___equal to___ some specified value.
- Note that the original statement could become the ___null hypothesis___, it could become the ___alternative hypothesis___, or it might not correspond exactly to either.

3) Compute the test statistic, z, and find the critical values, z*, and the P-value. Include a sketch that illustrates the situation.

The ___test statistic___ is a value used in making a decision about the null hypothesis, and it is found by converting the sample statistic to a score with the assumption that the null hypothesis is true.

Formula:
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

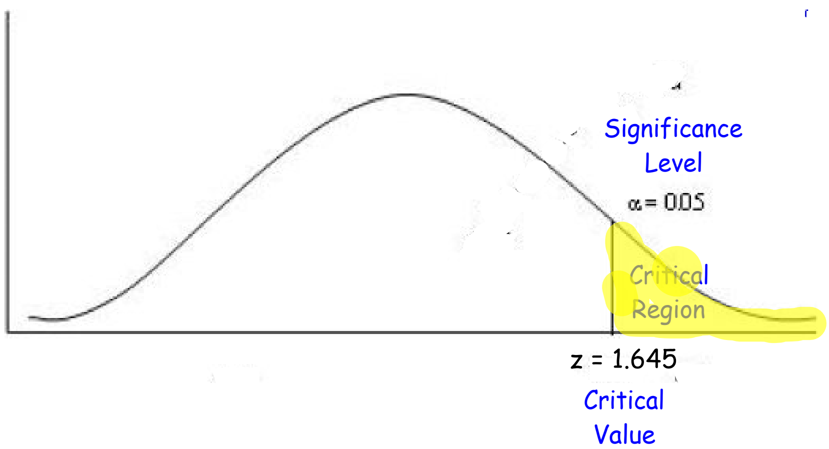
Critical Region, Significance Level, Critical Value, and P-Value

The __critical region__ (or rejection region) is the set of all values of the test statistic that causes us to reject the null hypothesis.

The _significance level_ (denoted by __α__) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true.

- If the test statistic falls in the critical region, we _reject_ the null hypothesis, so __α__ is the probability of making the mistake of rejecting the null hypothesis when it is true.

A ___critical value___ is any value that separates the critical region from the values of the test statistic that do not lead to rejection of the null hypothesis.



The ______P-value______ is the probability of getting a value of the test statistic that is at least as extreme as the one representing the sample data, assuming the null hypothesis is true.

4) Write a Conclusion. (Two Parts)

- Determine whether to reject or fail to reject the null hypothesis, linking the reason to the P-value or to the critical values.

- Tell what your conclusion means in the context of the situation.

Decisions and Conclusions

The decision to reject or fail to reject the null hypothesis can be made by using any of the following methods:

Traditional Method

- Reject H_0 if the test statistic falls within the critical region.
- Fail to reject H_0 if the test statistic does not fall within the critical region.

P-Value Method

- Reject H_0 if the p-value $\leq \alpha$ (where α is the significance level, such as 0.05)
- Fail to reject H_0 if the p-value $> \alpha$.

AP Exam - Hypothesis Test Rubric

1) Hypotheses
   *State Null Hypothesis
   *State Alternative Hypothesis (with proper tails)
   *Use correct notation (defined if non-standard)

2) Test
   *Check assumptions and conditions (not just list)
   *Specify the model
   *Name the test

3) Mechanics
   *Show work (statistics, values subbed into formula, shaded sketch of model, etc.)
   *Report test statistic (z, t, χ^2, df)
   *Report P-value

4) Conclusion
   *State decision (Reject/Fail to Reject)
   *With linkage to the P-value ("because the P-value is so low..." or something like that)
   *Interpret the result in context

<u>Example</u>:
Ships arriving in the US ports are inspected by Customs officials for contaminated cargo. Assume, for a certain port, 20% of the ships arriving in the previous year contained cargo that was contaminated. A random selection of 50 ships in the current year included 5 that had contaminated cargo. Does the data suggest that the proportion of ships arriving in the port with contaminated cargoes has decreased in the current year? Use $\alpha = 0.01$.

1 prop z test.

$$z = \frac{5/50 - .2}{\sqrt{\frac{(.2)(.8)}{50}}} = -1.77$$

Cond.
1. SRS
2. $50(.2) = 10 \geq 10$
   $50(.8) = 40 \geq 10$
3. $50 \times 10 = 500 <$ all ships w/ cargo

Ho: $p = .2$
Ha: $p < .2$
p = prop. of ships w/ cont. cargo

Trad

Fail to Reject Ho b/c test stat. outside critical region

P-value

$.0384 > .01$ fail to reject Ho b/c p-value > α level

Conc

There is insufficient evidence to support the claim that there is a decrease in the prop. of ships w/ cont. cargo.

Type I and Type II Errors

Decision	True State of Nature: The null hypothesis is true	True State of Nature: The null hypothesis is false
We decide to reject the null hypothesis	**TYPE I ERROR** (rejecting a true null hypothesis) α	Correct Decision
We fail to reject the null hypothesis	Correct Decision	**TYPE II ERROR** (failing to reject a false null hypothesis) β

Ways to Remember: RouTiNe FoR FuN

Type 1: Shawshank   Type 2: OJ Simpson

Example:
Identify the type I and type II error that corresponds to the given hypothesis:

The proportion of LHS students that are seniors is 0.27.

Type I: We reject the claim that the proportion of LHS students that are seniors is 0.27, when in fact, it is true.

Type II: We fail to reject the claim that the proportion of LHS students that are seniors is 0.27, when in fact, it is false.

Section 8.2 Notes

Type I Error:

- To decrease the probability of a Type I Error, make α smaller. Changing the sample size has no effect on the probability of a Type I error.
- If the null hypothesis is false, you can't make a Type I Error.

Type II Error:

- To decrease the probability of a Type II Error, take a larger sample or make α larger.
- If the null hypothesis is true, you can't make a Type II Error.

Power:

- Power: the probability of rejecting a null hypothesis.
- When the null hypothesis is false, you want to reject it and therefore you want the power to be large.
- To increase power, you can either take a larger sample or make α larger.